Solving Trigonometric Equations Using Identities

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Learning Objectives

- Apply Pythagorean identities to simplify trigonometric equations
- Solve trigonometric equations on the interval $[0, 2\pi)$
- Factor trigonometric expressions to find all solutions

Solve each trigonometric equation for θ on the interval [0, 2π), showing all identity substitutions and algebraic steps.

1. Solve for θ on [0, 2π).

$$\sin^2\theta - \cos^2\theta = 0$$

Answer: ______________________________

2. Solve for x on [0, 2π).

$$\cos^2 x + \cos x - \sin^2 x = 0$$

Answer: ______________________________

3. Solve for θ on [0, 2π).

$$2\sin^2\theta - 1 = 0$$

Answer: ______________________________

4. Solve for θ on [0, 2π).

$$2\cos^2\theta - \cos\theta - 1 = 0$$

Answer: ______________________________

5. Solve for θ on [0, 2π).

$$\sin^2\theta + \cos\theta = 1$$

Answer: ______________________________

6. Solve for x on [0, 2π).

$$2\cos^2 x + 3\cos x + 1 = 0$$

Answer: ______________________________

7. Solve for θ on [0, 2π).

$$\tan^2\theta - 1 = 0$$

Answer: ______________________________

8. Solve for θ on [0, 2π).

$$2\sin^2\theta + \sin\theta - 1 = 0$$

Answer: ____

9. Solve for x on [0, 2π).

$$\cos^2 x - \sin^2 x = \frac{1}{2}$$

Answer: ____

Encourage students to treat θ like a variable x and remind them to include ± when taking square roots and to rationalize radical denominators.

Solutions

1. Solve for θ on [0, 2π).

$\sin^2\theta - \cos^2\theta = 0$

→ *Replace sine squared theta with one minus cosine squared theta using the Pythagorean identity.*
→ *Combine like terms to get one minus two cosine squared theta equals zero.*
→ *Subtract one from both sides and divide by negative two to isolate cosine squared theta.*
→ *Take the square root of both sides, remembering plus or minus, then rationalize the denominator.*
→ *Use the unit circle to find the four angles where cosine theta equals plus or minus root two over two.*

Answer: $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

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2. Solve for x on [0, 2π).

$\cos^2 x + \cos x - \sin^2 x = 0$

→ *Replace sine squared x with one minus cosine squared x to make every term contain cosine.*
→ *Combine like terms to obtain two cosine squared x plus cosine x minus one equals zero.*
→ *Factor the quadratic in cosine x as two cosine x minus one times cosine x plus one.*
→ *Set each factor equal to zero to get cosine x equals one half or cosine x equals negative one.*
→ *Use the unit circle to identify the angles pi over three, five pi over three, and pi.*

Answer: $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

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3. Solve for θ on [0, 2π).

$2\sin^2\theta - 1 = 0$

→ *Add one to both sides and divide by two to isolate sine squared theta.*
→ *Take the square root of both sides and include the plus or minus sign.*
→ *Rationalize the denominator to get sine theta equals plus or minus root two over two.*
→ *Use the unit circle to list the four quadrantal solutions in [0, 2π).*

Answer: $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

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4. Solve for θ on [0, 2π).

$2\cos^2\theta - \cos\theta - 1 = 0$

→ *Treat cosine theta as a variable and factor the quadratic as two cosine theta plus one times cosine theta minus one.*
→ *Set each factor equal to zero to get cosine theta equals negative one half or cosine theta equals one.*
→ *Use the unit circle to find theta equals two pi over three and four pi over three for the first factor.*
→ *Identify theta equals zero as the solution where cosine theta equals one on the interval.*

Answer: $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

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5. Solve for θ on [0, 2π).

$\sin^2\theta + \cos\theta = 1$

→ *Replace sine squared theta with one minus cosine squared theta using the Pythagorean identity.*
→ *Simplify to get negative cosine squared theta plus cosine theta equals zero.*
→ *Factor out cosine theta to get cosine theta times one minus cosine theta equals zero.*
→ *Set each factor equal to zero, giving cosine theta equals zero or cosine theta equals one.*
→ *Use the unit circle to list theta equals zero, pi over two, and three pi over two.*

Answer: $\theta = 0, \frac{\pi}{2}, \frac{3\pi}{2}$

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6. Solve for x on [0, 2π).

$2\cos^2 x + 3\cos x + 1 = 0$

→ *Factor the quadratic in cosine x as two cosine x plus one times cosine x plus one.*
→ *Set each factor equal to zero to get cosine x equals negative one half or cosine x equals negative one.*
→ *Use the unit circle to find x equals two pi over three and four pi over three for the first factor.*
→ *Identify x equals pi as the solution where cosine x equals negative one.*

Answer: $x = \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

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7. Solve for θ on [0, 2π).

$\tan^2\theta - 1 = 0$

→ *Add one to both sides to obtain tangent squared theta equals one.*
→ *Take the square root of both sides and include the plus or minus sign.*
→ *Solve tangent theta equals one and tangent theta equals negative one separately.*
→ *Use the unit circle to list the four reference angles at pi over four in each quadrant.*

Answer: $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

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8. Solve for θ on [0, 2π).

$2\sin^2\theta + \sin\theta - 1 = 0$

→ *Factor the quadratic in sine theta as two sine theta minus one times sine theta plus one.*
→ *Set each factor equal to zero to get sine theta equals one half or sine theta equals negative one.*
→ *Use the unit circle to find theta equals pi over six and five pi over six for the first factor.*
→ *Identify theta equals three pi over two as the solution where sine theta equals negative one.*

Answer: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

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9. Solve for x on [0, 2π).

$\cos^2 x - \sin^2 x = \frac{1}{2}$

→ *Replace sine squared x with one minus cosine squared x using the Pythagorean identity.*
→ *Simplify to get two cosine squared x minus one equals one half.*
→ *Add one to both sides and divide by two to isolate cosine squared x.*
→ *Take the square root of both sides, include plus or minus, and rationalize to get cosine x equals plus or minus root three over two.*
→ *Use the unit circle to list all four solutions in [0, 2π).*

Answer: $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$