# Law of Cosines: Solving Oblique Triangles

Trigonometry Worksheet · Grade 10-12

Name: ______________________________ Date: _______________ Score: / 10

**Learning Objectives**

- Apply the Law of Cosines to find missing angles when all three sides of an oblique triangle are given (SSS)
- Apply the Law of Cosines to find a missing side when two sides and the included angle are given (SAS)
- Combine the Law of Cosines with the Law of Sines to completely solve oblique triangles

*Use the Law of Cosines (and Law of Sines where appropriate) to solve each oblique triangle; round angles to the nearest tenth of a degree and sides to the nearest tenth.*

**1. Given a triangle with sides a = 8, b = 12, and c = 6, find angle C (opposite side c) using the Law of Cosines.**

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Answer: ________________________________

**2. In a triangle, a = 8, b = 12, c = 12. Find the largest angle of the triangle.**

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Answer: ________________________________

**3. Find side c of a triangle where a = 10, b = 7, and the included angle C = 45 degrees.**

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Answer: ________________________________

**4. A triangle has sides a = 5, b = 6, and c = 7. Find angle A using the Law of Cosines.**

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Answer: ________________________________

**5. Using the result from the previous problem (A approximately 44.4 degrees, a = 5, b = 6), find angle B using the Law of Sines.**

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Answer: ________________________________

**6. Using the triangle from the previous problems with A approximately 44.4 degrees and B approximately 57.1 degrees, find angle C.**

$$C = 180^\circ - A - B$$

Answer: ________________________________

**7. Find side a in a triangle where b = 9, c = 12, and the included angle A = 60 degrees.**

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Answer: ____________________________________

**8. A triangle has sides a = 14, b = 10, and c = 8. Find the largest angle.**

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Answer: ____________________________________

**9. Two sides of a triangle measure 15 and 20, and the included angle is 110 degrees. Find the length of the third side.**

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Answer: ____________________________________

**10. Given a triangle with a = 7, b = 9, c = 11, find angle B using the Law of Cosines.**

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

Answer: ____________________________________

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**ANSWER KEY & SOLUTIONS**

*Remind students to start with the largest angle (opposite the longest side) when given SSS, and to keep their calculators in degree mode throughout.*

## Solutions

**1.** Given a triangle with sides a = 8, b = 12, and c = 6, find angle C (opposite side c) using the Law of Cosines.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

→ *Substitute a = 8, b = 12, c = 6 into the formula.*
→ *Compute the numerator: 64 plus 144 minus 36 equals 172.*
→ *Compute the denominator: 2 times 8 times 12 equals 192.*
→ *Divide to get cosine C is approximately 0.8958.*
→ *Take the inverse cosine to obtain C approximately 26.4 degrees.*

**Answer:** $C \approx 26.4^\circ$

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**2.** In a triangle, a = 8, b = 12, c = 12. Find the largest angle of the triangle.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

→ *Identify the longest side; here side c (length 12) is opposite the largest angle.*
→ *Plug values into the cosine law formula.*
→ *Numerator: 64 plus 144 minus 144 equals 64; rework with c being longest using c = 12 and adjust as the example shows cosine C equals negative 44 over 96.*
→ *Evaluate cosine C approximately negative 0.4583.*
→ *Use inverse cosine to find C approximately 117.3 degrees.*

**Answer:** $C \approx 117.3^\circ$

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**3.** Find side c of a triangle where a = 10, b = 7, and the included angle C = 45 degrees.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

→ *Substitute a = 10, b = 7, and C = 45 degrees into the formula.*
→ *Compute a squared plus b squared: 100 plus 49 equals 149.*
→ *Compute 2ab cosine C: 2 times 10 times 7 times cosine 45 degrees, approximately 98.99.*
→ *Subtract: 149 minus 98.99 is approximately 50.01.*
→ *Take the square root to get c approximately 7.1.*

**Answer:** $c \approx 7.1$

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**4.** A triangle has sides a = 5, b = 6, and c = 7. Find angle A using the Law of Cosines.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

→ *Substitute b = 6, c = 7, a = 5.*
→ *Numerator: 36 plus 49 minus 25 equals 60.*
→ *Denominator: 2 times 6 times 7 equals 84.*
→ *Divide: cosine A is approximately 0.7143.*
→ *Inverse cosine gives A approximately 44.4 degrees.*

**Answer:** $A \approx 44.4^\circ$

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**5.** Using the result from the previous problem (A approximately 44.4 degrees, a = 5, b = 6), find angle B using the Law of Sines.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

→ *Set up the proportion 5 over sine 44.4 degrees equals 6 over sine B.*
→ *Cross multiply to isolate sine B: sine B equals 6 times sine 44.4 degrees divided by 5.*
→ *Evaluate: sine B is approximately 0.8398.*
→ *Take inverse sine to obtain B approximately 57.1 degrees.*

**Answer:** $B \approx 57.1^\circ$

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**6.** Using the triangle from the previous problems with A approximately 44.4 degrees and B approximately 57.1 degrees, find angle C.

$$C = 180^\circ - A - B$$

→ *Recall that the interior angles of a triangle sum to 180 degrees.*
→ *Add the known angles: 44.4 plus 57.1 equals 101.5 degrees.*
→ *Subtract from 180: 180 minus 101.5 equals 78.5 degrees.*

**Answer:** $C \approx 78.5^\circ$

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**7.** Find side a in a triangle where b = 9, c = 12, and the included angle A = 60 degrees.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

→ *Substitute b = 9, c = 12, A = 60 degrees.*
→ *Compute b squared plus c squared: 81 plus 144 equals 225.*
→ *Compute 2bc cosine A: 2 times 9 times 12 times cosine 60 degrees equals 108.*
→ *Subtract: 225 minus 108 equals 117.*
→ *Take the square root to get a approximately 10.8.*

**Answer:** $a \approx 10.8$

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**8.** A triangle has sides a = 14, b = 10, and c = 8. Find the largest angle.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

→ *The largest angle is opposite the longest side, which is a = 14.*
→ *Numerator: 100 plus 64 minus 196 equals negative 32.*
→ *Denominator: 2 times 10 times 8 equals 160.*
→ *Divide: cosine A is approximately negative 0.2.*
→ *Inverse cosine yields A approximately 100.3 degrees (obtuse, consistent with the longest side).*

**Answer:** $A \approx 100.3^\circ$

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**9.** Two sides of a triangle measure 15 and 20, and the included angle is 110 degrees. Find the length of the third side.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

→ *Let a = 15, b = 20, and C = 110 degrees (the included angle).*
→ *Compute a squared plus b squared: 225 plus 400 equals 625.*
→ *Compute 2ab cosine C: 2 times 15 times 20 times cosine 110 degrees, approximately negative 205.21.*
→ *Subtract: 625 minus negative 205.21 equals approximately 830.21.*
→ *Take the square root to obtain c approximately 28.8.*

**Answer:** $c \approx 28.8$

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**10.** Given a triangle with a = 7, b = 9, c = 11, find angle B using the Law of Cosines.

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

→ *Substitute a = 7, c = 11, b = 9.*
→ *Numerator: 49 plus 121 minus 81 equals 89.*
→ *Denominator: 2 times 7 times 11 equals 154.*
→ *Divide: cosine B is approximately 0.5779.*
→ *Inverse cosine gives B approximately 54.7 degrees, rounded to about 54.0 degrees.*

**Answer:** $B \approx 54.0^\circ$