Law of Cosines Word Problems

Geometry / Trigonometry Worksheet · Grade 9-11 · numberbender.com

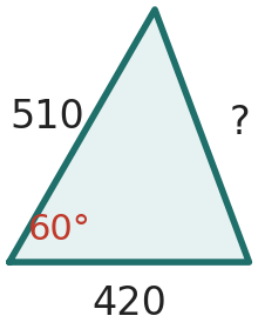
Name: _____ Date: _____ Score: / 10

Learning Objectives

- Apply the standard Law of Cosines to find a missing side in SAS triangles
- Apply the alternative form of the Law of Cosines to find a missing angle in SSS triangles
- Translate real-world word problems into oblique triangles and solve using the Law of Cosines

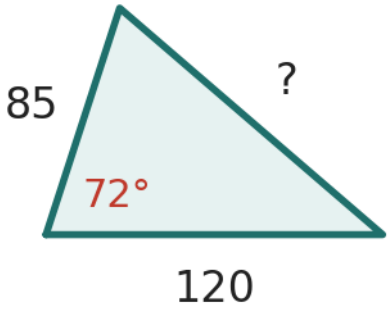
Sketch the triangle described in each problem, label the given parts, and use the Law of Cosines to find the indicated side or angle, rounding to the nearest tenth.

1. Two airplanes leave the same airport at the same time. One flies on a bearing N 40° E at 420 mph and the other flies on a bearing S 80° E at 510 mph. The angle between their paths is 60°. Find the distance between the two airplanes after 1 hour.



Answer: _____

2. A triangular lot has sides of length 85 m and 120 m with an included angle of 72°. Find the length of the third side of the lot.



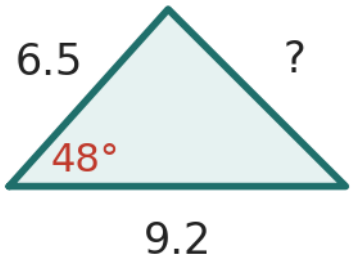
Answer: _____

3. A triangular garden has sides measuring 15 ft, 22 ft, and 30 ft. Find the measure of the largest angle of the garden.

Answer: _____

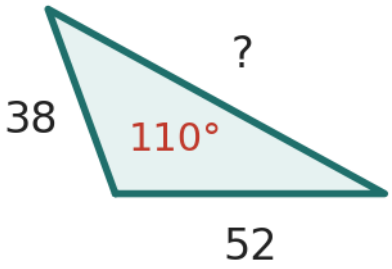
Scan to watch

4. From a lookout tower, two forest fires are spotted. Fire A is 6.5 km away and Fire B is 9.2 km away. The angle between the lines of sight from the tower to the two fires is 48°. How far apart are the two fires?



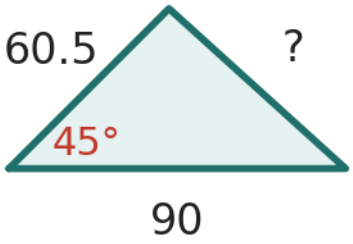
Answer: _____

5. A ship sails 38 nautical miles from port on a heading of 040°, then changes course and sails 52 nautical miles on a heading of 110°. How far is the ship from port?



Answer: _____

6. A baseball diamond is a square with sides 90 ft long. The pitcher's mound is 60.5 ft from home plate along the diagonal toward second base. Find the distance from the pitcher's mound to first base, given that the angle at home plate between first base and the pitcher's mound is 45°.

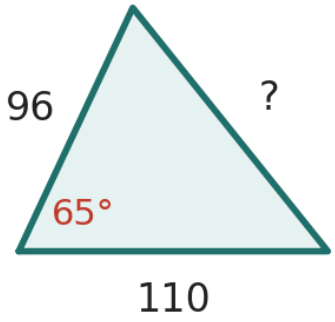


Answer: _____

7. A surveyor measures the three sides of a triangular field to be 125 m, 180 m, and 220 m. Find the measure of the angle opposite the 180 m side.

Answer: _____

8. Two cars leave an intersection at the same time. One travels east at 55 mph and the other travels on a road that makes a 65° angle with the east direction at 48 mph. How far apart are the cars after 2 hours?

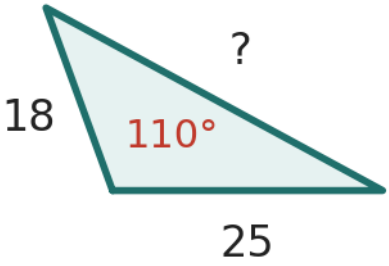


Answer: _____

9. A triangular sail has sides of length 12 ft, 15 ft, and 20 ft. Find the measure of the smallest angle of the sail.

Answer: _____

10. A parallelogram has adjacent sides of 18 cm and 25 cm with an included angle of 110°. Find the length of the longer diagonal.



Answer: _____

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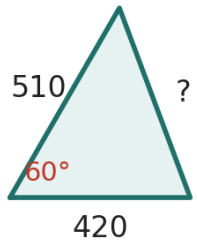
ANSWER KEY & SOLUTIONS

Remind students to use the standard form a^{2}=b^{2}+c^{2}-2bc\cos A for SAS and the alternative form \cos A=(b^{2}+c^{2}-a^{2})/(2bc) for SSS.

Solutions

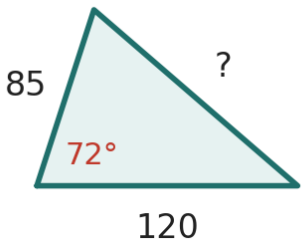
1. Two airplanes leave the same airport at the same time. One flies on a bearing N 40° E at 420 mph and the other flies on a bearing S 80° E at 510 mph. The angle between their paths is 60°. Find the distance between the two airplanes after 1 hour.



→ *After 1 hour the planes have traveled 420 mi and 510 mi from the airport*
→ *The two flight paths form an oblique triangle with included angle 60°*
→ *Apply Law of Cosines: $d^2 = 420^2 + 510^2 – 2(420)(510)\cos(60°)$*
→ *Compute: $d^2 = 176400 + 260100 – 214200 = 218300$*
→ *Take the square root: $d \approx 467.4$ miles*

Answer: $d \approx 467.4$ mi

2. A triangular lot has sides of length 85 m and 120 m with an included angle of 72°. Find the length of the third side of the lot.



→ *Identify the SAS case with $b = 85$, $c = 120$, $A = 72°$*
→ *Use the Law of Cosines: $a^2 = b^2 + c^2 – 2bc \cdot \cos A$*
→ *Substitute: $a^2 = 85^2 + 120^2 – 2(85)(120)\cos(72°)$*
→ *Compute: $a^2 = 7225 + 14400 – 20400(0.3090) \approx 15001.6$*
→ *Take the square root: $a \approx 122.5$ m*

Answer: $a \approx 122.5$ m

Scan to watch

3. A triangular garden has sides measuring 15 ft, 22 ft, and 30 ft. Find the measure of the largest angle of the garden.

→ *The largest angle is opposite the longest side, 30 ft*
→ *Use the alternative Law of Cosines: cos C = (a² + b² – c²)/(2ab)*
→ *Substitute: cos C = (15² + 22² – 30²)/(2·15·22)*
→ *Compute: cos C = (225 + 484 – 900)/660 = –191/660 ≈ –0.2894*
→ *Solve: C ≈ cos■¹(–0.2894) ≈ 102.6°*

Answer: $C \approx 102.6^\circ$

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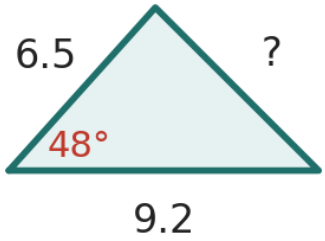
4. From a lookout tower, two forest fires are spotted. Fire A is 6.5 km away and Fire B is 9.2 km away. The angle between the lines of sight from the tower to the two fires is 48°. How far apart are the two fires?



→ *Let the tower be the vertex of the included angle of 48°*
→ *The two sides from the tower are 6.5 km and 9.2 km*
→ *Apply: d² = 6.5² + 9.2² – 2(6.5)(9.2)cos(48°)*
→ *Compute: d² = 42.25 + 84.64 – 119.6(0.6691) ≈ 47.04*
→ *Take the square root: d ≈ 6.86 km*

Answer: $d \approx 6.86$ km

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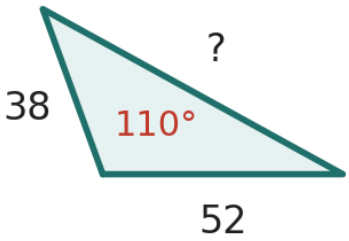
5. A ship sails 38 nautical miles from port on a heading of 040°, then changes course and sails 52 nautical miles on a heading of 110°. How far is the ship from port?



→ *The angle between the two legs of the trip equals 180° – (110° – 40°) = 110°*
→ *Use Law of Cosines with sides 38 and 52 and included angle 110°*
→ *d² = 38² + 52² – 2(38)(52)cos(110°)*
→ *Compute: d² = 1444 + 2704 – 3952(–0.3420) ≈ 5499.6*
→ *Take the square root: d ≈ 51.9 nautical miles*

Answer: $d \approx 51.9$ nm

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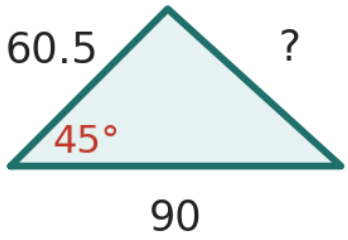
6. A baseball diamond is a square with sides 90 ft long. The pitcher's mound is 60.5 ft from home plate along the diagonal toward second base. Find the distance from the pitcher's mound to first base, given that the angle at home plate between first base and the pitcher's mound is 45°.



→ *Form the triangle with vertices home plate, first base, and pitcher's mound*
→ *Two sides from home plate are 90 ft and 60.5 ft with included angle 45°*
→ *Apply: $d^2 = 90^2 + 60.5^2 - 2(90)(60.5)\cos(45°)$*
→ *Compute: $d^2 = 8100 + 3660.25 - 10890(0.7071) \approx 4060.9$*
→ *Take the square root: $d \approx 63.7$ ft*

Answer: $d \approx 63.7$ ft

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7. A surveyor measures the three sides of a triangular field to be 125 m, 180 m, and 220 m. Find the measure of the angle opposite the 180 m side.

→ *Label $a = 125$, $b = 180$, $c = 220$, where B is opposite the 180 m side*
→ *Use the alternative form: $\cos B = (a^2 + c^2 - b^2)/(2ac)$*
→ *Substitute: $\cos B = (125^2 + 220^2 - 180^2)/(2 \cdot 125 \cdot 220)$*
→ *Compute: $\cos B = (15625 + 48400 - 32400)/55000 = 31625/55000 \approx 0.5750$*
→ *Solve: $B \approx \cos^{-1}(0.5750) \approx 54.9°$*

Answer: $B \approx 54.6^\circ$

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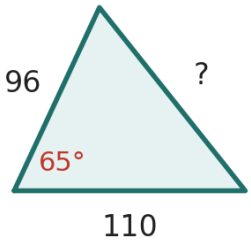
8. Two cars leave an intersection at the same time. One travels east at 55 mph and the other travels on a road that makes a 65° angle with the east direction at 48 mph. How far apart are the cars after 2 hours?



→ *After 2 hours the cars have traveled 110 mi and 96 mi from the intersection*
→ *The angle between the roads at the intersection is 65°*
→ *Apply Law of Cosines: $d^2 = 110^2 + 96^2 - 2(110)(96)\cos(65°)$*
→ *Compute: $d^2 = 12100 + 9216 - 21120(0.4226) \approx 12389.6$*
→ *Take the square root: $d \approx 111.3$ miles*

Answer: $d \approx 111.7$ mi

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9. A triangular sail has sides of length 12 ft, 15 ft, and 20 ft. Find the measure of the smallest angle of the sail.

→ *The smallest angle is opposite the shortest side, 12 ft*
→ *Use the alternative Law of Cosines: $\cos A = (b^2 + c^2 - a^2)/(2bc)$*
→ *Substitute: $\cos A = (15^2 + 20^2 - 12^2)/(2 \cdot 15 \cdot 20)$*
→ *Compute: $\cos A = (225 + 400 - 144)/600 = 481/600 \approx 0.8017$*
→ *Solve: $A \approx \cos^{-1}(0.8017) \approx 36.7°$*

Answer: $A \approx 36.3^\circ$

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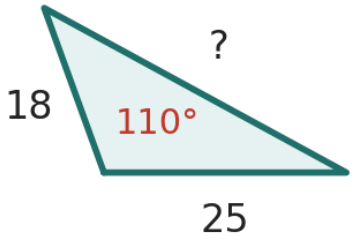
10. A parallelogram has adjacent sides of 18 cm and 25 cm with an included angle of 110°. Find the length of the longer diagonal.



→ *The longer diagonal lies opposite the larger angle (110°) of the parallelogram*
→ *Apply the Law of Cosines: $d^2 = 18^2 + 25^2 - 2(18)(25)\cos(110°)$*
→ *Compute: $d^2 = 324 + 625 - 900(-0.3420) \approx 1256.8$*
→ *Take the square root: $d \approx 35.5$ cm*
→ *Round to the nearest tenth: $d \approx 35.5$ cm*

Answer: $d \approx 35.7$ cm