Multiplying and Dividing Complex Numbers in Trigonometric Form

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Learning Objectives

- Multiply complex numbers expressed in trigonometric (polar) form
- Divide complex numbers expressed in trigonometric (polar) form
- Simplify trigonometric form results and convert to standard a+bi form when required

Perform the indicated operation on the complex numbers given in trigonometric form and simplify your answer.

1. Simplify the complex number into standard form a+bi.

$3(\cos 45^\circ + i\sin 45^\circ)$

Answer: _____

2. Simplify the complex number into standard form a+bi.

$\sqrt{3}(\cos 390^\circ + i\sin 390^\circ)$

Answer: _____

3. Let z1 = 4(cos 60° + i sin 60°) and z2 = 2(cos 30° + i sin 30°). Find z1·z2 in trigonometric form.

$z_1 \cdot z_2 = 4(\cos 60^\circ + i\sin 60^\circ) \cdot 2(\cos 30^\circ + i\sin 30^\circ)$

Answer: _____

4. Let z1 = 6(cos 120° + i sin 120°) and z2 = 3(cos 45° + i sin 45°). Find z1/z2 in trigonometric form.

$$\frac{z_1}{z_2} = \frac{6(\cos 120^\circ + i\sin 120^\circ)}{3(\cos 45^\circ + i\sin 45^\circ)}$$

Answer: _____

5. Multiply the complex numbers and express the result in trigonometric form.

$5(\cos 80^\circ + i\sin 80^\circ) \cdot 2(\cos 100^\circ + i\sin 100^\circ)$

Answer: _____

6. Divide the complex numbers and express the result in trigonometric form.

$$\frac{12(\cos 200^\circ + i\sin 200^\circ)}{4(\cos 110^\circ + i\sin 110^\circ)}$$

Answer: _____

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7. Multiply the complex numbers and simplify to standard form a+bi.

$$\sqrt{2}(\cos 135^\circ + i\sin 135^\circ) \cdot 3(\cos 45^\circ + i\sin 45^\circ)$$

Answer: _____

8. Divide the complex numbers and simplify to standard form a+bi.

$$\frac{8(\cos 210^\circ + i\sin 210^\circ)}{2(\cos 60^\circ + i\sin 60^\circ)}$$

Answer: _____

9. Find the product (z1)(z2) in trigonometric form, where z1 = 7(cos 25° + i sin 25°) and z2 = 4(cos 50° + i sin 50°).

$$z_1 \cdot z_2$$

Answer: _____

10. Find z1/z2 in trigonometric form, where z1 = 10(cos 300° + i sin 300°) and z2 = 5(cos 150° + i sin 150°).

$$\frac{z_1}{z_2}$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Remind students that when multiplying, moduli multiply and arguments add; when dividing, moduli divide and arguments subtract.

Solutions

1. Simplify the complex number into standard form a+bi.

$3(\cos 45^\circ + i\sin 45^\circ)$

→ *Substitute cos 45° = √2/2 and sin 45° = √2/2.*
→ *Distribute the 3 to each term to get 3(√2/2) + 3(√2/2)i.*
→ *Write the result as (3√2)/2 + (3√2)/2 i.*

Answer: $\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

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2. Simplify the complex number into standard form a+bi.

$\sqrt{3}(\cos 390^\circ + i\sin 390^\circ)$

→ *Reduce 390° by subtracting 360° to get a coterminal angle of 30°.*
→ *Substitute cos 30° = √3/2 and sin 30° = 1/2.*
→ *Multiply by √3 to get (√3)(√3/2) + (√3)(1/2)i = 3/2 + (√3/2)i.*

Answer: $\frac{3}{2} + \frac{\sqrt{3}}{2}i$

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3. Let z1 = 4(cos 60° + i sin 60°) and z2 = 2(cos 30° + i sin 30°). Find z1·z2 in trigonometric form.

$z_1 \cdot z_2 = 4(\cos 60^\circ + i\sin 60^\circ) \cdot 2(\cos 30^\circ + i\sin 30^\circ)$

→ *Multiply the moduli: 4 × 2 = 8.*
→ *Add the arguments: 60° + 30° = 90°.*
→ *Write the product as 8(cos 90° + i sin 90°).*

Answer: $8(\cos 90^\circ + i\sin 90^\circ)$

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4. Let z1 = 6(cos 120° + i sin 120°) and z2 = 3(cos 45° + i sin 45°). Find z1/z2 in trigonometric form.

$\frac{z_1}{z_2} = \frac{6(\cos 120^\circ + i\sin 120^\circ)}{3(\cos 45^\circ + i\sin 45^\circ)}$

→ *Divide the moduli: 6 ÷ 3 = 2.*
→ *Subtract the arguments: 120° – 45° = 75°.*
→ *Write the quotient as 2(cos 75° + i sin 75°).*

Answer: $2(\cos 75^\circ + i\sin 75^\circ)$

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5. Multiply the complex numbers and express the result in trigonometric form.

$5(\cos 80^\circ + i\sin 80^\circ) \cdot 2(\cos 100^\circ + i\sin 100^\circ)$

→ *Multiply the moduli: 5 × 2 = 10.*
→ *Add the arguments: 80° + 100° = 180°.*
→ *Write the product as 10(cos 180° + i sin 180°), which simplifies to –10.*

Answer: $10(\cos 180^\circ + i\sin 180^\circ)$

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6. Divide the complex numbers and express the result in trigonometric form.

$$\frac{12(\cos 200^\circ + i\sin 200^\circ)}{4(\cos 110^\circ + i\sin 110^\circ)}$$

→ *Divide the moduli: 12 ÷ 4 = 3.*
→ *Subtract the arguments: 200° – 110° = 90°.*
→ *Write the quotient as 3(cos 90° + i sin 90°), which simplifies to 3i.*

Answer: $3(\cos 90^\circ + i\sin 90^\circ)$

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7. Multiply the complex numbers and simplify to standard form a+bi.

$\sqrt{2}(\cos 135^\circ + i\sin 135^\circ) \cdot 3(\cos 45^\circ + i\sin 45^\circ)$

→ *Multiply the moduli: √2 × 3 = 3√2.*
→ *Add the arguments: 135° + 45° = 180°.*
→ *Evaluate: 3√2(cos 180° + i sin 180°) = 3√2(–1 + 0i) = –3√2.*

Answer: $-3\sqrt{2}$

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8. Divide the complex numbers and simplify to standard form a+bi.

$$\frac{8(\cos 210^\circ + i\sin 210^\circ)}{2(\cos 60^\circ + i\sin 60^\circ)}$$

→ *Divide the moduli: 8 ÷ 2 = 4.*
→ *Subtract the arguments: 210° – 60° = 150°.*
→ *Substitute cos 150° = –√3/2 and sin 150° = 1/2; multiply by 4 to get –2√3 + 2i… correct: 4(–√3/2) + 4(1/2)i = –2√3 + 2i.*

Answer: $-2\sqrt{3} - 2i$

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9. Find the product (z1)(z2) in trigonometric form, where z1 = 7(cos 25° + i sin 25°) and z2 = 4(cos 50° + i sin 50°).

$z_1 \cdot z_2$

→ *Multiply the moduli: 7 × 4 = 28.*
→ *Add the arguments: 25° + 50° = 75°.*
→ *Write the product as 28(cos 75° + i sin 75°).*

Answer: $28(\cos 75^\circ + i\sin 75^\circ)$

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10. Find z1/z2 in trigonometric form, where z1 = 10(cos 300° + i sin 300°) and z2 = 5(cos 150° + i sin 150°).

$\frac{z_1}{z_2}$

→ *Divide the moduli: 10 ÷ 5 = 2.*
→ *Subtract the arguments: 300° – 150° = 150°.*
→ *Write the quotient as 2(cos 150° + i sin 150°).*

Answer: $2(\cos 150^\circ + i\sin 150^\circ)$