Complex Numbers in Trigonometric Form: Conversion, Multiplication, and Di

Pre-Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Convert complex numbers from trigonometric form to standard form using unit circle values
- Multiply two complex numbers in trigonometric form by multiplying moduli and adding arguments
- Divide two complex numbers in trigonometric form by dividing moduli and subtracting arguments

Convert each complex number to standard form a + bi, or perform the indicated multiplication or division using the trigonometric form formulas.

1. Convert the complex number to standard form.

$3(\cos 45^\circ + i\sin 45^\circ)$

Answer: _____

2. Convert the complex number to standard form.

$\sqrt{3}(\cos 390^\circ + i\sin 390^\circ)$

Answer: _____

3. Convert the complex number to standard form.

$4(\cos 60^\circ + i\sin 60^\circ)$

Answer: _____

4. Convert the complex number to standard form.

$6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Answer: _____

5. Convert the complex number to standard form.

$2(\cos 480^\circ + i\sin 480^\circ)$

Answer: _____

6. Multiply the two complex numbers in trigonometric form. Leave your answer in trigonometric form.

$z_1 \cdot z_2 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \cdot 3\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

Answer: _____

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7. Multiply the two complex numbers in trigonometric form and convert to standard form.

$4(\cos 30^\circ + i\sin 30^\circ) \cdot 5(\cos 60^\circ + i\sin 60^\circ)$

Answer: _____

8. Divide the two complex numbers in trigonometric form. Leave the answer in trigonometric form.

$$\frac{12(\cos 150^\circ + i\sin 150^\circ)}{3(\cos 30^\circ + i\sin 30^\circ)}$$

Answer: _____

9. Divide the two complex numbers and convert the quotient to standard form.

$$\frac{10\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)}{2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}$$

Answer: _____

10. Multiply the two complex numbers and convert the product to standard form.

$\sqrt{2}(\cos 45^\circ + i\sin 45^\circ) \cdot \sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$

Answer: _____

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ANSWER KEY & SOLUTIONS

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Remind students to reduce angles greater than 360 degrees (or 2π) before applying unit circle values, and to simplify radicals in their final answers.

Solutions

1. Convert the complex number to standard form.

$3(\cos 45^\circ + i\sin 45^\circ)$

→ *Substitute the unit circle values: cosine of 45 degrees equals square root of 2 over 2, and sine of 45 degrees equals square root of 2 over 2.*
→ *Distribute the 3 to both terms inside the parentheses.*
→ *Write the result as a real part plus an imaginary part.*

Answer: $\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

2. Convert the complex number to standard form.

$\sqrt{3}(\cos 390^\circ + i\sin 390^\circ)$

→ *Subtract 360 degrees from 390 degrees to get a coterminal angle of 30 degrees within the unit circle range.*
→ *Replace cosine of 30 degrees with square root of 3 over 2 and sine of 30 degrees with one half.*
→ *Distribute the square root of 3 across both terms, simplifying square root of 3 times square root of 3 to 3.*

Answer: $\frac{3}{2} + \frac{\sqrt{3}}{2}i$

3. Convert the complex number to standard form.

$4(\cos 60^\circ + i\sin 60^\circ)$

→ *Replace cosine of 60 degrees with one half and sine of 60 degrees with square root of 3 over 2.*
→ *Distribute the 4 across each term inside the parentheses.*
→ *Simplify each product to obtain the real and imaginary parts.*

Answer: $2 + 2\sqrt{3}\,i$

4. Convert the complex number to standard form.

$6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

→ *Recall that pi over 3 radians equals 60 degrees on the unit circle.*
→ *Substitute cosine of pi over 3 with one half and sine of pi over 3 with square root of 3 over 2.*
→ *Distribute the 6 and simplify each term.*

Answer: $3 + 3\sqrt{3}\,i$

Scan to watch

5. Convert the complex number to standard form.

$2(\cos 480^\circ + i\sin 480^\circ)$

→ *Subtract 360 degrees from 480 degrees to obtain the coterminal angle of 120 degrees.*
→ *Use the unit circle: cosine of 120 degrees is negative one half and sine of 120 degrees is square root of 3 over 2.*
→ *Distribute the 2 across both terms to reach standard form.*

Answer: $-1 + \sqrt{3}\,i$

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6. Multiply the two complex numbers in trigonometric form. Leave your answer in trigonometric form.

$z_1 \cdot z_2 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \cdot 3\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

→ *Multiply the moduli: 2 times 3 equals 6.*
→ *Add the arguments: 2 pi over 3 plus 11 pi over 6 equals 15 pi over 6, which simplifies to 5 pi over 2.*
→ *Reduce 5 pi over 2 by subtracting 2 pi to get the equivalent angle of pi over 2.*

Answer: $6\left(\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right) = 6\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

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7. Multiply the two complex numbers in trigonometric form and convert to standard form.

$4(\cos 30^\circ + i\sin 30^\circ) \cdot 5(\cos 60^\circ + i\sin 60^\circ)$

→ *Multiply the moduli: 4 times 5 equals 20.*
→ *Add the arguments: 30 degrees plus 60 degrees equals 90 degrees.*
→ *Apply the unit circle: cosine of 90 degrees is 0 and sine of 90 degrees is 1, so the product becomes 20 times i.*

Answer: $20i$

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8. Divide the two complex numbers in trigonometric form. Leave the answer in trigonometric form.

$\frac{12(\cos 150^\circ + i\sin 150^\circ)}{3(\cos 30^\circ + i\sin 30^\circ)}$

→ *Divide the moduli: 12 divided by 3 equals 4.*
→ *Subtract the arguments: 150 degrees minus 30 degrees equals 120 degrees.*
→ *Write the quotient in the form r times the quantity cosine theta plus i sine theta.*

Answer: $4(\cos 120^\circ + i\sin 120^\circ)$

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9. Divide the two complex numbers and convert the quotient to standard form.

$\frac{10\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)}{2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}$

→ *Divide the moduli: 10 divided by 2 equals 5.*
→ *Subtract the arguments: 5 pi over 4 minus pi over 4 equals pi.*
→ *Apply the unit circle: cosine of pi is negative 1 and sine of pi is 0, so the quotient simplifies to negative 5.*

Answer: -5

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10. Multiply the two complex numbers and convert the product to standard form.

$\sqrt{2}(\cos 45^\circ + i\sin 45^\circ) \cdot \sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$

→ *Multiply the moduli: square root of 2 times square root of 2 equals 2.*

→ *Add the arguments: 45 degrees plus 135 degrees equals 180 degrees.*

→ *Apply the unit circle: cosine of 180 degrees is negative 1 and sine of 180 degrees is 0, so the product simplifies to negative 2.*

Answer: -2