De Moivre's Theorem

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Learning Objectives

- Convert complex numbers from rectangular form to trigonometric (polar) form
- Apply De Moivre's Theorem to raise complex numbers to integer powers
- Express the final result in rectangular form a + bi

Use De Moivre's Theorem to evaluate each power of a complex number and express your final answer in rectangular form.

1. Evaluate the following using De Moivre's Theorem.

$(-1 + i\sqrt{3})^{12}$

Answer: ______________________

2. Use De Moivre's Theorem to evaluate.

$(1 + i)^5$

Answer: ______________________

3. Evaluate using De Moivre's Theorem.

$(1 - i)^8$

Answer: ______________________

4. Evaluate using De Moivre's Theorem.

$(\sqrt{3} + i)^6$

Answer: ______________________

5. Use De Moivre's Theorem to simplify.

$[2(\cos 40^\circ + i \sin 40^\circ)]^3$

Answer: ______________________

6. Evaluate using De Moivre's Theorem.

$(-1 - i)^4$

Answer: ______________________

7. Use De Moivre's Theorem to evaluate.

$(2 + 2i)^6$

Answer: ______________________

8. Evaluate the power using De Moivre's Theorem.

$[3(\cos 60^\circ + i\sin 60^\circ)]^4$

Answer: ____________________________________

9. Evaluate using De Moivre's Theorem.

$(\sqrt{3} - i)^5$

Answer: ____________________________________

10. Use De Moivre's Theorem to evaluate.

$(1 + i\sqrt{3})^{10}$

Answer: ____________________________________

Remind students to first convert to trigonometric form z = r(cos θ + i sin θ) before applying z^{n} = r^{n}(cos nθ + i sin nθ).

Solutions

1. Evaluate the following using De Moivre's Theorem.

$(-1 + i\sqrt{3})^{12}$

→ *Find r = sqrt((-1)² + (sqrt 3)²) = sqrt(1 + 3) = 2.*
→ *Find θ: the point (-1, sqrt 3) lies in quadrant II, so θ = 120°.*
→ *Write in trig form: 2(cos 120° + i sin 120°).*
→ *Apply De Moivre: 2^12 (cos(12·120°) + i sin(12·120°)) = 4096(cos 1440° + i sin 1440°).*
→ *Reduce 1440° mod 360° = 0°, so the value is 4096(1 + 0i) = 4096.*

Answer: 4096

2. Use De Moivre's Theorem to evaluate.

$(1 + i)^5$

→ *Find r = sqrt(1² + 1²) = sqrt 2.*
→ *Find θ = 45° since the point (1, 1) is in quadrant I.*
→ *Write in trig form: sqrt 2 (cos 45° + i sin 45°).*
→ *Apply De Moivre: (sqrt 2)^5 (cos 225° + i sin 225°) = 4·sqrt 2 (cos 225° + i sin 225°).*
→ *Substitute cos 225° = -sqrt 2/2 and sin 225° = -sqrt 2/2 to get 4·sqrt 2 · (-sqrt 2/2 - i·sqrt 2/2) = -4 - 4i.*

Answer: $-4 - 4i$

3. Evaluate using De Moivre's Theorem.

$(1 - i)^8$

→ *Compute r = sqrt(1 + 1) = sqrt 2.*
→ *Find θ = -45° (or 315°) since (1, -1) is in quadrant IV.*
→ *Trig form: sqrt 2 (cos(-45°) + i sin(-45°)).*
→ *Apply De Moivre: (sqrt 2)^8 (cos(-360°) + i sin(-360°)) = 16(1 + 0i).*
→ *Final answer is 16.*

Answer: 16

4. Evaluate using De Moivre's Theorem.

$(\sqrt{3} + i)^6$

→ *Compute r = sqrt(3 + 1) = 2.*
→ *Find θ = 30° since the point (sqrt 3, 1) is in quadrant I.*
→ *Trig form: 2(cos 30° + i sin 30°).*
→ *Apply De Moivre: 2^6 (cos 180° + i sin 180°) = 64(-1 + 0i).*
→ *Final answer: -64.*

Answer: -64

5. Use De Moivre's Theorem to simplify.

$[2(\cos 40^\circ + i\sin 40^\circ)]^3$

→ *Raise the modulus to the power: 2^3 = 8.*
→ *Multiply the angle by the exponent: 3 × 40° = 120°.*
→ *Write: 8(cos 120° + i sin 120°).*
→ *Substitute cos 120° = -1/2 and sin 120° = sqrt 3/2.*
→ *Multiply: 8(-1/2) + 8i(sqrt 3/2) = -4 + 4i·sqrt 3.*

Answer: $8(\cos 120^\circ + i\sin 120^\circ) = -4 + 4i\sqrt{3}$

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6. Evaluate using De Moivre's Theorem.

$(-1 - i)^4$

→ *Compute r = sqrt(1 + 1) = sqrt 2.*
→ *The point (-1, -1) is in quadrant III, so θ = 225°.*
→ *Trig form: sqrt 2 (cos 225° + i sin 225°).*
→ *Apply De Moivre: (sqrt 2)^4 (cos 900° + i sin 900°) = 4(cos 180° + i sin 180°).*
→ *Substitute: 4(-1 + 0i) = -4.*

Answer: -4

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7. Use De Moivre's Theorem to evaluate.

$(2 + 2i)^6$

→ *Compute r = sqrt(4 + 4) = 2·sqrt 2.*
→ *Find θ = 45° since the point (2, 2) is in quadrant I.*
→ *Trig form: 2·sqrt 2 (cos 45° + i sin 45°).*
→ *Apply De Moivre: (2·sqrt 2)^6 (cos 270° + i sin 270°) = 512(0 - i) = -512i.*

Answer: $-512i$

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8. Evaluate the power using De Moivre's Theorem.

$[3(\cos 60^\circ + i\sin 60^\circ)]^4$

→ *Raise the modulus: 3^4 = 81.*
→ *Multiply the angle: 4 × 60° = 240°.*
→ *Write: 81(cos 240° + i sin 240°).*
→ *Substitute cos 240° = -1/2 and sin 240° = -sqrt 3/2.*
→ *Multiply: 81(-1/2) + 81i(-sqrt 3/2) = -81/2 - (81·sqrt 3/2)i.*

Answer: $-\frac{81}{2} - \frac{81\sqrt{3}}{2}i$

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9. Evaluate using De Moivre's Theorem.

$(\sqrt{3} - i)^5$

→ *Compute r = sqrt(3 + 1) = 2.*
→ *The point (sqrt 3, -1) is in quadrant IV, so θ = -30°.*
→ *Trig form: 2(cos(-30°) + i sin(-30°)).*
→ *Apply De Moivre: 2^5 (cos(-150°) + i sin(-150°)) = 32(-sqrt 3/2 - i/2).*
→ *Multiply: -16·sqrt 3 - 16i.*

Answer: $-16\sqrt{3} - 16i$

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10. Use De Moivre's Theorem to evaluate.

$(1 + i\sqrt{3})^{10}$

→ *Compute r = sqrt(1 + 3) = 2.*
→ *Find θ = 60° since the point (1, sqrt 3) is in quadrant I.*
→ *Trig form: 2(cos 60° + i sin 60°).*
→ *Apply De Moivre: 2^10 (cos 600° + i sin 600°) = 1024(cos 240° + i sin 240°).*
→ *Substitute cos 240° = -1/2 and sin 240° = -sqrt 3/2: 1024(-1/2 - i·sqrt 3/2) = -512 - 512i·sqrt 3.*

Answer: $-512 - 512i\sqrt{3}$